

Testing the seesaw mechanism at collider energies in the Randall-Sundrum model

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Abstract

The Randall-Sundrum model with gauge fields and fermions in the bulk has several attractive features including the gauge coupling unification, a candidate for the dark matter, and an explanation for the hierarchical Yukawa couplings.

In this paper, we point out that the 1st Kaluza-Klein modes of right-handed neutrinos may be produced at colliders, for both Dirac and Majorana neutrino cases. Furthermore, we can see whether the neutrino masses are Dirac type or Majorana type from the mass spectrum of KK particles.

1 Introduction

Recently, models with extra dimensions were suggested as solutions to the gauge hierarchy problem. Amongst these models, the Randall-Sundrum (RS) model [1] attracted particular attention. In the RS model, the hierarchy between the electroweak and the Planck scales is explained by a warped extra dimension. Originally, all Standard Model (SM) fields are localized on the TeV brane. However, it was realized that it is sufficient to localize only the Higgs on the TeV brane to solve the hierarchy problem. Moreover, placing gauge fields and fermions in the bulk brings several attractive features: unification of the gauge couplings at high scale [2]–[8], a candidate for the dark matter [9, 10], and a new interpretation for the hierarchical Yukawa couplings [11, 12].

The hierarchical Yukawa couplings are explained by the overlaps of the Higgs and fermions. Recently, certain types of configurations of fermions are obtained [13] that reproduce not only the fermions mass matrices, but also satisfy the Flavor Changing Neutral Current (FCNC) constraints for $m_A^{(1)} \geq 1$ TeV¹. These models also sufficiently suppress unwanted non-renormalizable operators except for B and L breaking ones.

In a recent work [14], we showed that we can not suppress the operators for B and L breaking by the configurations of fermions that reproduce the realistic mass matrices. These unwanted operators are easily suppressed by discrete gauge symmetries [15], but it is then expected that the seesaw mechanism [16] does not work. However, we found that the seesaw mechanism does work to explain the observed small mass of neutrinos, if L is broken on the Planck brane [14].

In this paper, we point out that the 1st Kaluza-Klein (KK) modes of right-handed neutrinos may be produced at e^+e^- colliders, for both Dirac and Majorana neutrino cases. In 4D theories, it is difficult to produce right-handed neutrinos at colliders, since they have large masses ($\sim 10^{15}$ GeV) or small Yukawa couplings ($\sim 10^{-12}$). In the RS model, KK fermions have masses of order TeV and Yukawa couplings of order 1, which enables the production of KK right-handed neutrinos at colliders. Furthermore, we can determine the bulk configurations of fermions by comparing the masses of KK gauge bosons and KK fermions. Thus we can see whether the neutrino masses are Dirac type or Majorana type from the mass spectrum of KK particles.

¹ Here $m_A^{(1)}$ denotes the mass of the 1st KK mode of gauge boson (see Appendix).

2 Setup

The metric of the RS model is

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad (1)$$

where $\sigma = k|y|$, and $k \sim M_P \sim 10^{18}$ GeV is the AdS curvature. The fifth dimension y is compactified on an orbifold S^1/Z_2 . Two 3-branes reside at the fixed points $y = 0$ and $y = \pi R$, which are referred to as the Planck brane and the TeV brane, respectively.

We assume that the SM gauge bosons and fermions are in the bulk, and Higgs is on the TeV brane. The kinetic term of Higgs is

$$\begin{aligned} S_{\text{Higgs}} &= \int d^4x dy \sqrt{g_{MN}} g^{\mu\nu} D_\mu H(x) D_\nu H(x) \delta(y - \pi R) \\ &= \int d^4x \eta^{\mu\nu} D_\mu \tilde{H} D_\nu \tilde{H}, \end{aligned} \quad (2)$$

where $\tilde{H} = e^{-k\pi R} H$ is a canonically normalized field. Thus we have $\langle H \rangle = e^{k\pi R} v / \sqrt{2}$.

The hierarchical Yukawa couplings are explained by the configurations of fermions. For example, Yukawa interaction terms of the charged leptons are

$$\begin{aligned} S_{\text{Yukawa}} &= \int d^4x dy \sqrt{g_{MN}} (\lambda_{e5})_{ij} H \bar{\ell}_i(x, y) e_j(x, y) \delta(y - \pi R) \\ &= \int d^4x (\lambda_{e5})_{ij} T_m(c_{\ell i}) T_n(c_{ej}) \tilde{H} \bar{\ell}_i^{(m)}(x) e_j^{(n)}(x), \end{aligned} \quad (3)$$

where $\ell_i^{(m)}$, $e_j^{(n)}$ are KK modes of leptons, and $T_m(c_{\ell i})$, $T_n(c_{ej})$ are their couplings on the TeV brane (see Appendix). The lower suffixes i, j denote generations of electroweak eigenstates, which are often omitted in the following. The zero modes correspond to SM leptons. With a moderate tuning of the $O(1)$ mass parameters c_ℓ and c_e , the hierarchical Yukawa couplings are reproduced by effective 4D couplings $\lambda_{e5} T_0(c_\ell) T_0(c_e)$.

We introduce a right-handed neutrino N in the bulk. The Yukawa interaction terms $H^* \bar{\ell} N$ generate Dirac masses for neutrinos. In the case of Majorana neutrino, N has a Majorana mass term (which breaks L) on the Planck brane,

$$\begin{aligned} S_{\text{Majorana}} &= \int d^4x dy \sqrt{g_{MN}} \lambda'_5 M(x) N^T(x, y) \gamma_2 N(x, y) \delta(y) \\ &= \int d^4x \lambda'_5 P_m(c_N) P_n(c_N) M N^{(m)T}(x) \gamma_2 N^{(n)}(x), \end{aligned} \quad (4)$$

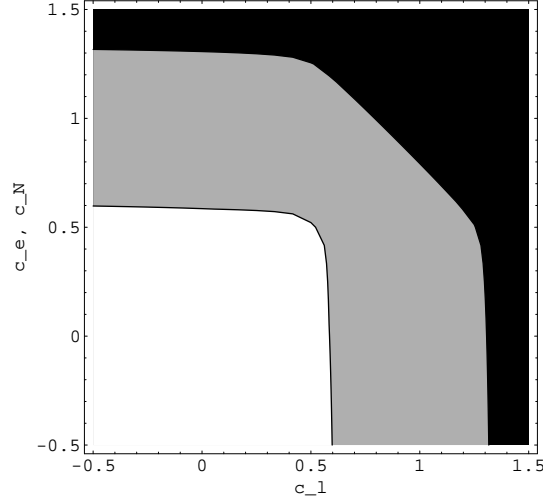


Figure 1: The Dirac masses of zero modes. The contours in the figure correspond to m_τ (bottom-left) and 5×10^{-2} eV (top-right). The first contour is plotted for c_ℓ and c_e , while the second one is for c_ℓ and c_N . Here we used $\lambda_{e5}, \lambda_{\nu5} = 1/k$.

where M is a 5D Majorana mass. Thus we obtain 4D Lagrangians

$$\mathcal{L}_{KK} = m_N^{(n)} \bar{N}^{(n)} N^{(n)} + m_\ell^{(n)} \bar{\ell}^{(n)} \ell^{(n)} + m_e^{(n)} \bar{e}^{(n)} e^{(n)}, \quad (5)$$

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= \lambda_{e5} T_m(c_\ell) T_n(c_e) \tilde{H} \bar{\ell}^{(m)} e^{(n)} \\ &\quad + \lambda_{\nu5} T_m(c_\ell) T_n(c_N) \tilde{H}^* \bar{\ell}^{(m)} N^{(n)}, \end{aligned} \quad (6)$$

$$\mathcal{L}_{\text{Majorana}} = \lambda' P_m(c_N) P_n(c_N) M N^{(m)T} \gamma_2 N^{(n)}. \quad (7)$$

3 Neutrino mass

In this section, we search for the parameters that reproduce the heaviest left-handed neutrino mass $\sqrt{\Delta m_{\text{atm}}^2} \simeq 5 \times 10^{-2}$ eV. We use $k = 2.4 \times 10^{18}$ GeV and $e^{k\pi R} = 2 \times 10^{15}$, which give KK gauge boson masses $m_A^{(1)} \simeq 3$ TeV, the lower bound from electroweak measurements [17]. We assume that 5D Yukawa couplings are $\lambda_{e5}, \lambda_{\nu5}, \lambda' \sim 1/k$, which correspond to 4D Yukawa couplings $\lambda_4 \sim 1$ for $c < 1/2$. We further assume that λ_{e5} is diagonal for simplicity.

We have to specify the value of $c_{\ell3}$ to evaluate the neutrino mass, since ℓ_3 has the largest Yukawa coupling to N . There is an upper bound on $c_{\ell3}$ from the τ mass. The

Dirac masses of zero modes are shown in figure 1. We see that $c_{\ell 3} \leq 0.6$ is necessary to reproduce the τ mass.

First, we consider Dirac neutrino case. In this case, N should be localized toward the Planck brane to obtain a small Yukawa coupling $\sim 10^{-12}$. From figure 1 we see that $c_N \simeq 1.3$ is required to reproduce the neutrino mass.

Next, we consider Majorana neutrino case. The mass matrix of the Majorana mass term of N is

$$\begin{bmatrix} MP_0(c_N)P_0(c_N) & MP_0(c_N)P_1(c_N) & MP_0(c_N)P_2(c_N) & \cdots \\ MP_1(c_N)P_0(c_N) & MP_1(c_N)P_1(c_N) & \cdots & \cdots \\ MP_2(c_N)P_0(c_N) & \vdots & \ddots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad (8)$$

which is diagonalized to give the eigenvalues

$$\sum_{n=0} MP_n(c_N)P_n(c_N), 0, 0, \cdots. \quad (9)$$

We assume $c_N > 1/2$ to ensure that $MP_0(c_N)P_0(c_N) \gg MP_n(c_N)P_n(c_N)$. In this case, mixings between the zero mode and the KK modes are small, and the KK modes are almost Dirac particles.

The effective left-handed neutrino mass is generated by both the seesaw mechanism and 1-loop effects, which are shown in figure 2. The KK modes do not contribute to the seesaw mechanism, since they are Dirac particles. The effective mass obtained from the seesaw mechanism is

$$m_{\text{seesaw}} = \frac{[\lambda_{\nu 5} T_0(c_\ell) T_0(c_N) \tilde{H}]^2}{\lambda' P_0(c_N)^2 M}. \quad (10)$$

The contribution from 1-loop effects is

$$m_{\text{loop}} \sim \frac{1}{16\pi^2} \frac{\pi^2 [\lambda_{\nu 5} T_0(c_\ell) T_1(c_N) \tilde{H}]^2 [\lambda' P_1(c_N)^2 M]}{m_1^2}, \quad (11)$$

where we have summed over all the KK modes under the approximations

$$P_n(c_N) = -(-1)^n P_1(c_N), \quad (12)$$

$$T_n(c_N) = T_1(c_N), \quad (13)$$

$$m_N^{(n)} = n m_N^{(1)}, \quad (14)$$

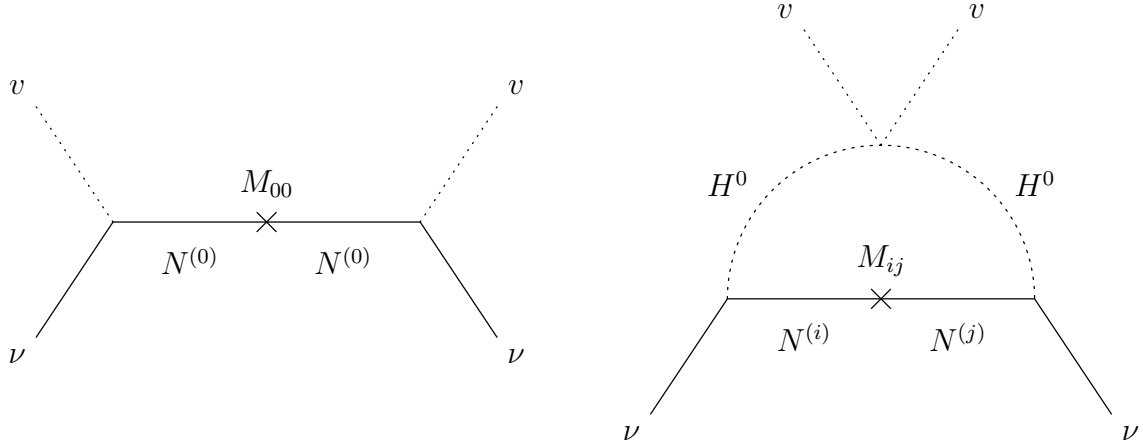


Figure 2: The diagrams that contribute to the effective neutrino mass. Here we use the notation $M_{ij} = \lambda' P_i(c_N) P_j(c_N) M$.

which are confirmed numerically.

The sum of these two contributions is plotted in figure 3. The effective neutrino mass is determined by the seesaw mechanism for $M < 10^5$ GeV, and by 1-loop effects for $M > 10^5$ GeV. The parameter c_N can be as low as 0.7, which is in contrast with $c_N \simeq 1.3$ for Dirac neutrino case.

Now we discuss the relation between the mass spectrum of the KK modes and configurations of fermions. The mass of the 1st KK mode of fermion is

$$m_\psi^{(1)} \simeq 2.4k e^{-k\pi R} [1 + 0.6|c - 1/2|], \quad (15)$$

and the mass for $c = 1/2$ is almost equal to the KK gauge boson mass $m_A^{(1)}$. Thus we can evaluate the c parameter from the ratio $m_\psi^{(1)}/m_A^{(1)}$. We showed that for Dirac neutrino case $c_N \simeq 1.3$, while for Majorana neutrino case $c_N \geq 0.7$. Hence we can see whether neutrino masses are Dirac type or Majorana type from measurements of the KK masses $m_A^{(1)}$ and $m_N^{(1)}$.

4 Collider signal

In 4D theories, the production of right-handed neutrinos at colliders is difficult, since their masses are large (for Majorana case), or their Yukawa couplings are small (for both

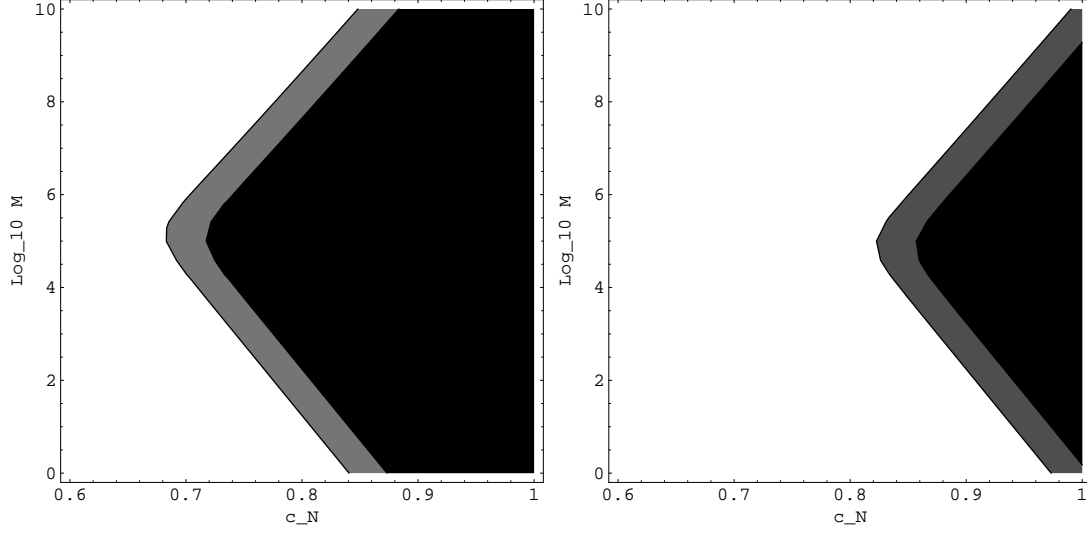


Figure 3: The effective neutrino mass. The figure on the left is for $c_{l3} = 0.6$ (upper bound from τ mass), and that on the right for $c_{l3} = -0.5$. The two contours correspond to 10^{-1} eV (left) and 10^{-2} eV (right).

Dirac and Majorana cases). In the RS model, the 1st KK modes of fermions have masses of order TeV, and Yukawa couplings of order 1. Thus the 1st KK mode of right-handed neutrino $N^{(1)}$ may be produced at TeV colliders, whether the neutrino mass is Dirac or Majorana type. In the following, we discuss how to produce $N^{(1)}$ for two cases, $c_{l1} < 1/2$ and $c_{l1} > 1/2$.

4.1 $c_{l1} < 1/2$

In this case, the Yukawa coupling between $N^{(1)}$ and electron is

$$\lambda = \lambda_{\nu 5} T_0(c_{l1}) T_1(c_N) \simeq \sqrt{1/2 - c_{l1}} \sim 1. \quad (16)$$

Thus $N^{(1)}$ can be produced by the Yukawa interactions. The diagrams of corresponding processes are shown in figure 4. The reaction $e^- e^+ \rightarrow N^{(1)} \bar{N}^{(1)}$ has the largest cross section

$$\sigma(e^- e^+ \rightarrow N^{(1)} \bar{N}^{(1)}) = 110 \text{ fb} \times \left(\frac{\lambda}{1.0} \right)^4 \left(\frac{6 \text{ TeV}}{E_{\text{cm}}} \right)^2 f(E_{\text{cm}}/2m_N^{(1)}), \quad (17)$$

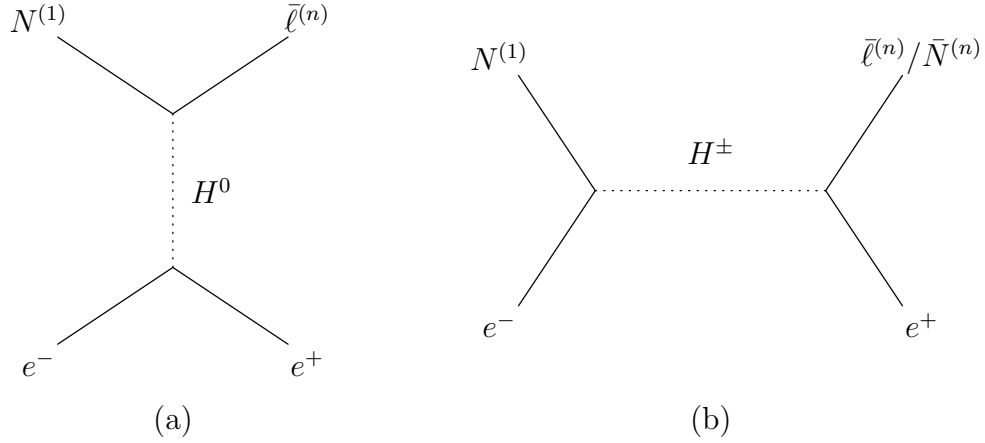


Figure 4: The diagrams of processes including $N^{(1)}$ in the final state.

where $f(x)$ is shown in figure 5. The reaction $e^-e^+ \rightarrow N^{(1)}\bar{N}^{(0)}$ is negligible, since it is suppressed by the Yukawa coupling between $N^{(0)}$ and electron

$$\lambda_{\nu 5} T_0(c_{\ell 1}) T_0(c_N) \simeq \sqrt{1/2 - c_{\ell 1}} \sqrt{c_N - 1/2} e^{(1/2 - c_N)k\pi R} \ll 1, \quad (18)$$

for both Dirac and Majorana neutrino cases. The processes (a) are suppressed by the electron Yukawa coupling $m_e/v \sim 10^{-6}$. The processes (b) with final states $N^{(1)}\bar{\ell}^{(n)}$ are also suppressed by m_e/v , since the chiralities of initial electrons are e_L^- and e_R^+ .

The decay modes of $N^{(1)}$ depend on c_N , $c_{\ell i}$ and c_{ei} , which determine the couplings and the KK masses of leptons. We consider the simplest case $m_N^{(1)} < m_{\ell i}^{(1)}$. The only decay mode of $N^{(1)}$ is

$$N^{(1)} \rightarrow \ell_i^{(0)} + H. \quad (19)$$

Thus we observe a lepton and a Higgs both with energy $\sim E_{\text{cm}}/4$, which is specific to the decay of $N^{(1)}$.

4.2 $c_{\ell 1} > 1/2$

In this case, the Yukawa coupling between $N^{(1)}$ and electron is

$$\lambda_5 T_0(c_{\ell 1}) T_1(c_N) \simeq \sqrt{c_{\ell 1} - 1/2} e^{(1/2 - c_{\ell 1})k\pi R} \ll 1. \quad (20)$$

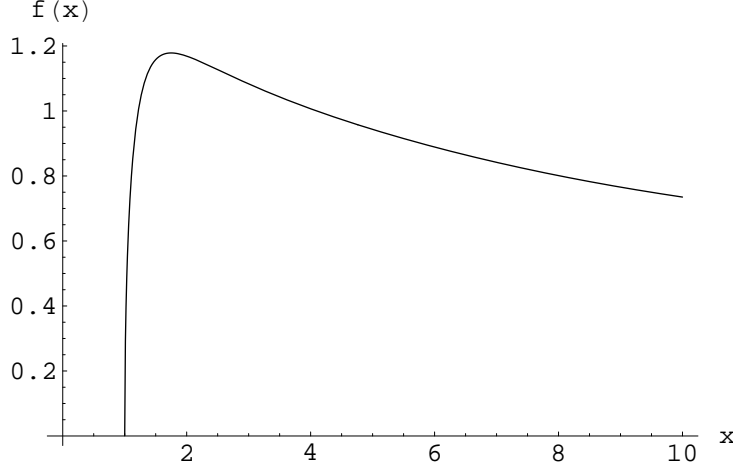


Figure 5: The function $f(x)$ of (17).

Thus we can not produce $N^{(1)}$ by the Yukawa interactions. However, the 1st KK modes of lepton doublets $\ell_i^{(1)}$ are produced by the processes of figure 6. Assuming the mass ordering $m_N^{(1)} < m_{\ell_i}^{(1)} < m_{ei}^{(1)}$, the main decay mode of $\ell_i^{(1)}$ is

$$\ell_i^{(1)} \rightarrow N^{(1)} + H, \quad (21)$$

since the Yukawa couplings between $\ell_i^{(1)}$ and $N^{(1)}$ are $\lambda_{\nu 5} \sqrt{k} \sqrt{k} \sim 1$.

For the case $c_{e1} < 1/2$, the Yukawa couplings between $\ell_1^{(1)}$ and electron are order 1. Thus the processes

$$e^- e^+ \rightarrow \ell_1^{(1)} \bar{\ell}_1^{(n)} \quad (n \geq 1) \quad (22)$$

occur with considerable rate, while other processes are suppressed by gauge couplings or the electron Yukawa coupling. The processes (22) with $n \geq 2$ are kinematically suppressed. The cross section of $e^- e^+ \rightarrow \ell_1^{(1)} \bar{\ell}_1^{(1)}$ is almost equal to (17).

For the case $c_{e1} > 1/2$, the Yukawa couplings between the KK modes of leptons and electron are much smaller than 1. Thus $\ell_i^{(1)}$ are produced by electroweak interactions (processes (e) and (f)) with cross sections of 1 – 10 fb for $E_{\text{cm}} \sim 6$ TeV, depending on the configurations and the KK masses of fermions.

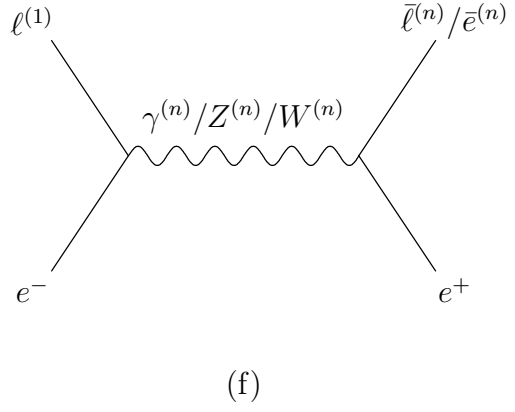
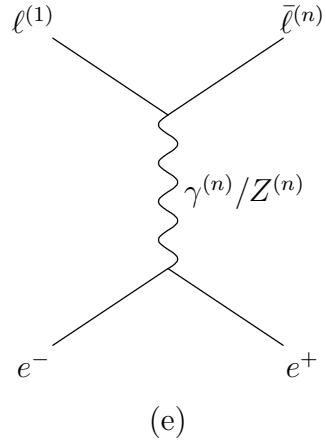
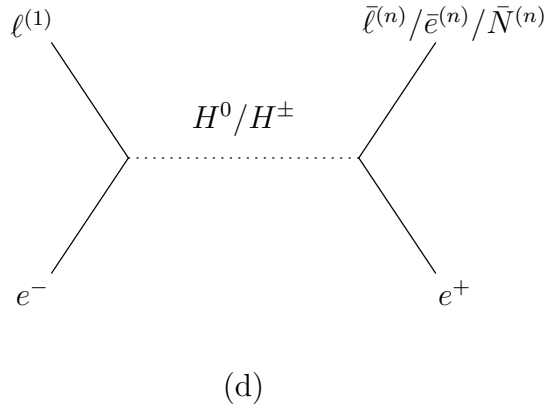
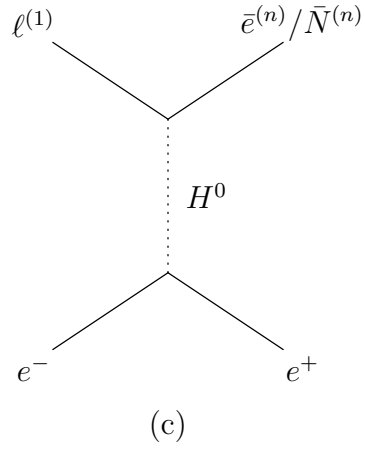


Figure 6: The diagrams of processes including $\ell^{(1)}$ in the final state.

5 Conclusion

We have shown in section 4 that the 1st KK mode of right-handed neutrino $N^{(1)}$ can be produced at e^+e^- colliders with $E_{\text{cm}} > 6$ TeV. In particular, if the left-handed electron $\ell_1^{(0)}$ is localized toward the TeV brane, then $N^{(1)}$ is produced with a considerable rate. From the measurement of masses of KK gauge bosons and $N^{(1)}$, we can see wheather neutrinos are Dirac or Majorana particles, as we discussed in section 3. This is a remarkable feature in the RS model, since in 4D theories it is difficult to distinguish the type of neutrino mass, which is related to the origin of matter in the universe [18].

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Appendix

We work in the gauge where

$$0 = A_5(x, y), \quad (23)$$

$$0 = \eta^{\mu\nu} \partial_\mu A_\nu(x, y). \quad (24)$$

In this gauge, the zero mode transforms as a gauge field, while the KK modes transform as adjoint fields. The gauge covariant derivative is

$$\begin{aligned} D_M &= \partial_M + ig_5 A_M(x, y) \\ &= \partial_M + ig_4 A_\mu^{(0)}(x) + (\text{KK modes}), \end{aligned} \quad (25)$$

where $g_5 = g_4 \sqrt{2\pi R}$ [19, 20].

The 5D fermions are decomposed to the KK modes [21],

$$\Psi(x, y) = \begin{bmatrix} \psi_L^{(n)}(x) f_L^{(n)}(y) \\ \psi_R^{(n)}(x) f_R^{(n)}(y) \end{bmatrix} = e^{(3/2)ky} \begin{bmatrix} \psi_L^{(n)}(x) \hat{f}_L^{(n)}(y) \\ \psi_R^{(n)}(x) \hat{f}_R^{(n)}(y) \end{bmatrix}, \quad (26)$$

and $\hat{f}_L^{(n)}(y)$, $\hat{f}_R^{(n)}(y)$ satisfies the normalization condition,

$$\delta_{mn} = \int_{-\pi R}^{\pi R} dy \hat{f}_L^{(m)}(y) \hat{f}_L^{(n)}(y) = \int_{-\pi R}^{\pi R} dy \hat{f}_R^{(m)}(y) \hat{f}_R^{(n)}(y). \quad (27)$$

The Z_2 transformation is given by $\Psi(x, -y) = \pm \gamma_5 \Psi(x, y)$. This property ensures that the zero modes are chiral. The KK modes form Dirac fields,

$$\psi^{(n)}(x) = \begin{bmatrix} \psi_L^{(n)}(x) \\ \psi_R^{(n)}(x) \end{bmatrix}. \quad (28)$$

We denote the profile of the n th even component as $f^{(n)}(y)$: $f^{(n)}(y) = f_L^{(n)}(y)$ or $f_R^{(n)}(y)$, depending on whether the zero mode is left- or right-handed. We introduce functions $P_n(c)$ and $T_n(c)$ defined by

$$P_n(c) \equiv \hat{f}^{(n)}(0), \quad (29)$$

$$T_n(c) \equiv \hat{f}^{(n)}(\pi R). \quad (30)$$

The 5D configuration depends on the bulk mass m_{bulk} . The configuration of the zero mode is

$$\hat{f}^{(0)}(y) = \sqrt{\frac{(1/2 - c)k}{e^{(1-2c)k\pi R} - 1}} e^{(1/2-c)ky} \quad (31)$$

$$\simeq \begin{cases} \sqrt{(1/2 - c)k} e^{(1/2-c)k(y-\pi R)} & \text{for } c < 1/2, \\ \sqrt{1/2\pi R} e^{(1/2-c)ky} & \text{for } c \simeq 1/2, \\ \sqrt{(c - 1/2)k} e^{(1/2-c)ky} & \text{for } c > 1/2, \end{cases} \quad (32)$$

where $c = m_{\text{bulk}}/k$. The couplings of the 1st KK mode on the branes are

$$T_1(c) \simeq \sqrt{k}, \quad (33)$$

$$P_1(c) \simeq \begin{cases} (1.4 - 2c)P_0(c) & \text{for } c < 1/2, \\ (0.6 - 2c)T_0(c) & \text{for } c > 1/2. \end{cases} \quad (34)$$

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